Vidyamandir Classes

Gurukul for IITJEE & Medical Preparation

SOLUTIONS

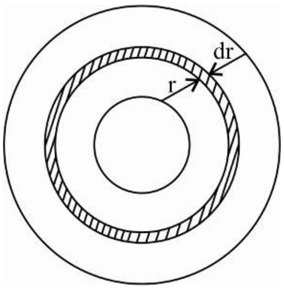
Joint Entrance Exam | IITJEE-2019

12th APRIL 2019 | Morning Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(3)



$$I = \int dmr^2$$

$$\Rightarrow \quad \int_a^b \left(\frac{\sigma_0}{r} 2\pi r\, dr\right) r^2 \quad \Rightarrow \quad I = 2\sigma_0 \pi \frac{(b^3 - a^3)}{3} \quad \Rightarrow \quad I = mk^2$$

$$\Rightarrow \quad 2\sigma_0 \pi \frac{(b^3 - a^3)}{3} = 2\sigma_0 \pi (b-a) K^2 \quad \Rightarrow \quad k = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

2.(1) $T = 2\pi\sqrt{\frac{I}{MH}} \Rightarrow H \propto \frac{1}{T^2}$

$$H_1 = B_1 \cos 45° = \frac{B_1}{\sqrt{2}}$$

$$H_2 = B_2 \cos 30° = \frac{\sqrt{3} B_2}{2} \qquad \therefore \qquad \frac{H_1}{H_2} = \left(\frac{T_2}{T_1}\right)^2 = \frac{B_1}{B_2} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \left(\frac{30}{40}\right)^2$$

$$\therefore \qquad \frac{B_1}{B_2} = \frac{9\sqrt{6}}{32} = 0.7$$

3.(3) $U = U_1 + U_2$

$$(n_1 + n_2) Cv \Delta T = n_1 Cv_1 \Delta T + n_2 Cv_2 \Delta T$$

$$Cv = \frac{n_1 Cv_1 - 1 + n_2 Cv_2}{n_1 + n_2} = \frac{2 \times \frac{3}{2} R + 3 \times \frac{5}{2} R}{5} = 2.1R$$

4.(3) $A = 0, B = 0, Y = 1$

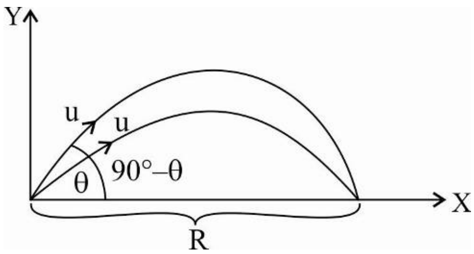
$A = 0, B = 1, Y = 1$

$A = 1, B = 0, Y = 0$

$A = 1, B = 1, Y = 0$

$\therefore$ option (3)

5.(3)



$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin(90°-\theta)}{g}$$

$$t_1 \cdot t_2 = \frac{2u\sin\theta}{g} \cdot \frac{2u\sin(90°-\theta)}{g}$$

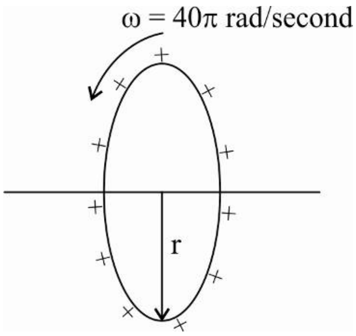
$$t_1 t_2 = \frac{2\times 2u^2 \sin\theta\cos\theta}{g^2}$$

$$t_1 t_2 = \frac{2u^2}{g} \cdot \frac{2\sin\theta\cos\theta}{g}$$

$$t_1 t_2 = \frac{2u^2}{g} \cdot \frac{\sin 2\theta}{g}$$

$$\boxed{t_1 t_2 = \frac{2R}{g}}$$

6.(1)



$$\omega = \frac{2\pi}{T}$$

$$I = \frac{q}{T}$$

$$I = \frac{q}{2\pi/\omega}$$

$$I = \frac{q\omega}{2\pi}$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\alpha$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\cdot 2\pi$$

$$B = \frac{\mu_0}{4\pi}\cdot 2\pi\frac{I}{r}$$

$$3.8\times10^{-9} = 10^{-7}\times 2\pi\times\frac{q\omega}{2\pi r}$$

$$3.8\times10^{-9} = 10^{-7}\times q\times\frac{40\pi}{10\times10^{-2}}$$

$$q = \frac{3.8\times10^{-9}\times10\times10^{-2}}{40\pi\times10^{-7}}$$

$$q = \frac{3.8\times10^{-10}\times10^{7}}{40\times\frac{22}{7}}$$

$$q = \frac{3.8\times10^{-3}\times7}{40\times22}$$

$$q = 0.030\times10^{-3}\,C$$

$$\boxed{q = 3\times10^{-5}\,C}$$

7.(4) $\Delta u_{ca} = -180$

$\Delta U_{ac} = 180$

For abc

$-\Delta Q = \Delta U + W$

$310 = 180 + W$

$W = 130\,J$

8.(2) $V = IR$

$$R = \frac{V}{I}$$

$$R = \frac{w/q}{I}$$

$$R = \frac{ML^2T^{-2}}{I^2T}$$

$$R = [ML^2T^{-3}A^{-2}]$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\varepsilon_0 = \frac{q_1q_2}{F_1r^2}$$

$$\varepsilon_0 = \frac{A^2T^2}{MLT^{-2}L^2}$$

$$\varepsilon_0 = \frac{A^2T^2}{ML^3T^{-2}}$$

$$\varepsilon_0 = [M^{-1}L^{-3}T^4A^2]$$

$$C^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$\mu_0 = \frac{1}{C^2\varepsilon_0}$$

$$\mu_0 = \frac{1}{L^2T^{-2}M^{-1}L^{-3}T^4A^2}$$

$$\mu_0 = \frac{M^1A^{-2}LT^{-2}}{}$$

$$\frac{\varepsilon_0}{\mu_0} = \frac{M^{-1}L^{-3}T^4A^2}{M^1A^{-2}LT^{-2}}$$

$$\frac{\varepsilon_0}{\mu_0} = M^{-2}L^{-4}T^6A^4$$

$$\frac{\mu_0}{\varepsilon_0} = M^2L^4T^{-6}A^{-4}$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = ML^2T^{-3}A^{-2}$$

9.(4) $W = M(g+a)l$

$a = \omega^2 L$

$\frac{1}{2}M\omega^2L^2 = Mgl(1-\cos\theta_0)$

$\omega^2 L = 2g(1-\cos\theta_0)$

$W = M[g+2g(1-\cos\theta_0)]l$

$W = M[g+2g-2g\cos\theta_0]l$

$W = mg[3-2\cos\theta_0]l$

$$W = mg\left[3-2\left(1-\frac{\theta_0^2}{2}\right)\right]l$$

$$W = Mg\left[3-2+2\times\frac{\theta_0^2}{2}\right]l$$

$W = Mg[1+\theta_0^2]l$

$W = Mgl[1+\theta_0^2]$

10.(4) $V_C \to$ velocity of child (Rightward)

$V_m \to$ velocity of man (Left ward)

$V_{C/M} = 0.7$ (take Rightward as positive)

$V_C - (-V_M) = 0.7$

$V_C = 0.7 - V_m$

Conservation of Linear momentum $p_i = p_t$

$0 = -50V_m + 20(0.7 - V_m)$

$V_m = 0.2$

11.(3) $y = A\sin(kx - \omega t + \phi)$

$V_p = -A\omega\cos(kx - \omega + \phi)$

At $t = 0, x = 0$

$V_p = -A\omega\cos\phi$

By diagram

$V_p = +ve$

$\cos\phi = -ve$

$\phi = \pi$

12.(1) $y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/\ell}$

$$y = \frac{mg}{A\alpha\,\Delta T}$$

$$m = \frac{yA\alpha\,\Delta T}{g} = 2\pi$$

13.(1) Case – I

(B) (A)

Source Observer

$$v^1 = v\left[\frac{1500-5}{1500-7.5}\right]$$

Case – II

(B) (A)

Observer Source

$$v^{11} = v^1\left[\frac{1500+7.5}{1500+5}\right] \approx 502\ Hr$$

14.(2) $R_i = 100\Omega$

$R_{output} = 100\times10^3\Omega$

Voltage gain = $\dfrac{\text{change in output voltage}}{\text{change in input voltage}}$

Voltage gain $= \dfrac{\Delta V_0}{\Delta V_i}$

$= \dfrac{\Delta I_C \times R_0}{\Delta I_b \times R_i}$

$A_v = 5\times10^4$

Power gain $= \dfrac{\text{change in output power}}{\text{change in input power}} = \dfrac{\Delta V_0\cdot\Delta I_C}{\Delta V_i\cdot\Delta I_b} = \dfrac{\Delta I_C\cdot R_0\cdot\Delta I_C}{\Delta I_b\cdot R_i\times\Delta I_b} = \left(\dfrac{\Delta I_C}{\Delta b}\right)^2\times\dfrac{R_0}{R_i}$

$= \left(\dfrac{5\times10^{-3}}{100\times10^{-6}}\right)^2\times\dfrac{100\times10^3}{100} = \dfrac{25\times10^{-6}}{10^{-8}}\times10^3$

Power gain $= 25\times10^5 = 2.5\times10^6$

15.(1) $\dfrac{1}{f} = \dfrac{1}{u}+\dfrac{1}{v}$

$\dfrac{1}{20} = \dfrac{1}{5}+\dfrac{1}{v}$

$v = \dfrac{-20}{3}\ (virtual)$

$d_{app} = \dfrac{d_{real}}{\mu} = \dfrac{\left(\dfrac{20}{3}+5\right)}{4/3}$

$d_{app} = 8.8\ cm$

16.(2) $i_g = 20\times50 = 1000/4\ Amp$

For $0-2v$

$i_g\ (R+r_1) = 2$

$r_1 = 1900\Omega = R_1$

For $0-10V$

$i_g(R+r_2)=10$

$r_2=9900\Omega$

$R_2=9900-1900=8000\Omega$

For $0-20V$

$i_g(R+r_3)=20$

$r_3=1900D\Omega$

$R_3=r_3-r_2=10000\Omega$

17.(2) Total energy

$$E=\left(\frac{1240}{108.5}+\frac{1240}{30.4}\right)eV$$

$$13.6\times Z^2\left(1-\frac{1}{n^2}\right)=E \quad \therefore \quad 13.6\times 4\left(1-\frac{1}{n^2}\right)=1240\left(\frac{138.9}{108.5\times 30.4}\right) \quad \therefore \quad n=5$$

18.(2) Optical path difference $=(\mu-1)t$

Shift $=\beta=\dfrac{\lambda D}{d}$

$$\therefore \quad (\mu-1)t=\beta\cdot\frac{d}{D} \Rightarrow \quad (\mu-1)t=\lambda \quad \therefore \quad t=\frac{\lambda}{\mu-1}$$

19.(4) $I=0, V=1.5V=\in$

$\therefore \quad \in=1.5V$

$I=1000\,mA=1A$

$V\approx 0$

$\Rightarrow \quad \in-Ir=0 \quad \Rightarrow \quad 1.5-1\times r=0$

$r=1.5\Omega$

20.(3) $E_1=\dfrac{1}{2}C_1V^2, E_2=\dfrac{1}{2}C_2V^2$

$$\frac{1}{C_1}=\frac{d/3}{K_1\in_0 A}+\frac{d/3}{K_2C_oA}+\frac{d/3}{K_3\in_0 A}$$

$$C_1=\frac{3\in_0 A}{d}\left(\frac{k_1k_2k_3}{k_1k_2+k_2k_3+k_3k_4}\right)$$

$$C_2=\frac{A}{3\in_0 d}\left(k_1+k_2+k_3\right) \quad \therefore \quad \frac{E_1}{E_2}=\frac{C_1}{C_2}=\frac{9k_1\,k_2k_3}{(k_1+k_2+k_3)\,(k_1k_2+k_3-k_2+k_3k_1)}$$

21.(2) Equivalent resistance $=2R+R+4R+R=8R$

$$P=\frac{\varepsilon^2}{8R}=4$$

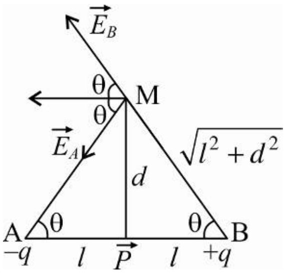
$$\Rightarrow \quad \frac{16\times 16}{8R}=4\Rightarrow R=8\Omega$$

22.(3) $W=h\nu_0=6.63\times 10^{-34}\times 10^{14}\times 4J$

$$W=6.63\times 10^{20}\,J\times 4 \;=\frac{4\times 6.63\times 10^{-20}}{1.6\times 10^{-19}}eV=1.66\;eV$$

23.(3) Electrostatic shielding

24.(2) $\vec{P} = -P_0\hat{x}$



$V_M = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{\sqrt{l^2+d^2}} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{l^2+d^2}}$, $\cos\theta = \frac{l}{\sqrt{l^2+d^2}}$

$V_M = 0$

For short dipole at equatorial position

$\vec{E}_M = \frac{-\vec{P}}{4\pi\varepsilon_0 d^3}$

25.(4) $Q_{gain} = Q_{lost}$

$M_1 \times 0.5 \times 10 + M_1 L = M_2 \times 1 \times 50$

$L = \frac{M_2}{M_1} \times 50 - 5$

26.(1) $R \cdot P = \frac{0.61\lambda}{\mu \sin\theta} = 0.61 \times \frac{\lambda}{N.A.}$

$N.A = \mu \sin\theta = 1.25$

$R.P = 0.61 \times \frac{5\times10^{-7}}{1.25} = 0.24\ \mu m$

27.(1) by comparing

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta = 2$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\frac{g}{2u^2\cos^2\theta} = 9$

$u = \frac{5}{3}$

28.(3) $\frac{1}{R_1} = \frac{1}{4} + \frac{1}{2}$, $R_{eq} = R_1 + 1.7 = \frac{4}{3} + 1.7$

$\frac{1}{R_1} = \frac{1+2}{4}$, $R_{eq} = \frac{4+5.1}{3}$

$R_1 = \frac{4}{3}$

$$R_{eq} = \frac{9.1}{3}$$

$$\varepsilon = Bvl$$

$$\varepsilon = 1 \times \frac{1 \times 10^{-2}}{1} \times 5 \times 10^{-2}$$

$$\varepsilon = 5 \times 10^{-4} \, volt$$

$$\varepsilon = IR_{eq}$$

$$I = \frac{V}{R_{eq}}$$

$$I = \frac{5 \times 10^{-4}}{\frac{9.1}{3}} = \frac{15 \times 10^{-4}}{9.1}$$

$$I = 1.648 \times 10^{-4}$$

$$I = 164.8 \times 10^{-6}$$

$$I = 164.8 \, \mu A$$

29.(4) $\vec{S} = -6\hat{j} + 8\hat{k}$

$$\hat{S} = \frac{\vec{S}}{|\vec{S}|}$$

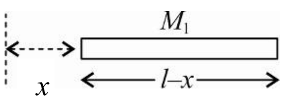
$$\hat{S} = \frac{6\hat{j} - 8\hat{k}}{\sqrt{(6)^2 + (-8)^2}}$$

$$\hat{S} = \frac{6\hat{j} - 8\hat{k}}{\sqrt{100}}$$

$$\hat{S} = \frac{2(3\hat{j} - 4\hat{k})}{10}$$

$$\hat{S} = \frac{3\hat{j} - 4\hat{k}}{5}$$

30.(3)



$$M^1 = \frac{M}{l}(l - x)$$

$$T_x = M^1 \omega^2 r$$

$$T_x = \frac{M}{l}(l - x)(\omega^2)\left(x + \frac{l - x}{2}\right)$$

$$T_x = \frac{m\omega^2 (l^2 - x^2)}{2l}$$

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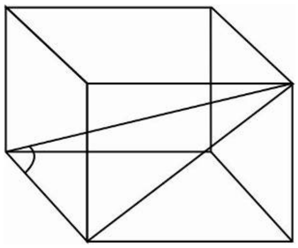
PART-B	CHEMISTRY

1. (3) $Na_3PO_4 + 3HNO_3 \rightarrow H_3PO_4 + 3NaNO_3$

$H_3PO_4 + 12(NH_4)_2 MoO_4 + 21\,HNO_3 \rightarrow (NH_4)_3 PO_4.12\,MoO_3$ (Yellow ppt)

2.(1) Due to formation of $p\pi - d\pi$ back bonding. $N(SiH_3)$ is Sp^2 hybridized. Hence it is planar and less basic than $N(CH_3)_3$.

3.(2) $\cos\theta = \frac{1}{\sqrt{3}}$



The distance of corner atom to teat heel a void along diagonal = $\frac{\sqrt{3}a}{4}$

Component along any side = $\frac{\sqrt{3}a}{4} \times \frac{1}{\sqrt{3}} = \frac{a}{4}$

$\left(a - \frac{a}{4} + \frac{a}{4}\right) = \frac{a}{2}$

4.(4) Cu^+ ion converted to Cu^{2+} ion and Cu^0.

5.(1) Probability of finding electron is maximum at region a and c

6.(1) $K_{sp} = [s][0.2]^3$

$[s] = 24 \times 10^{-24} \Big/ [0.2]^3$

$= 3 \times 10^{-22}$

7.(1) $m = \frac{X_{solution} X\,1000}{X_{solvent} X \text{ molar mass of solvent}} = 13.88$

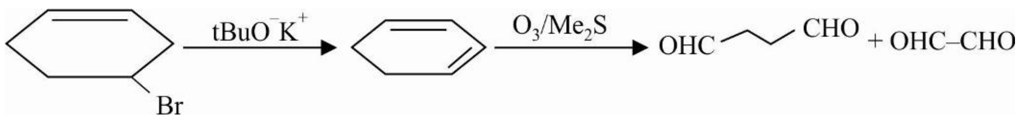
8.(1) $W = -P_{ext.}\Delta v$

$= -0.987 \times 9 = -8.8$ Latm

$1\text{Latm} = 101.3\text{J}$

$-8.8\text{L atm} = -0.9\text{kJ}$

9.(3)



10.(2) $I_{2(s)} \longrightarrow I_2(g)$

$\Delta H_2 = \Delta H_1 + \Delta Cp(T_2 - T_1)$

$\Delta C_p = C_{p(I_2(g))} - C_{p(I_2(S))} = 0.055 - 0.031 = 0.024 \text{ cal g}^{-1}\text{ k}^{-1}$

$\Delta H_2 = 24 - 0.024 \times 50 = 22.8 \text{cal g}^{-1}\text{k}^{-1}$

11.(1) Bakelite is a thermosetting polymer.

12.(1) $\left[\text{Fe}(\text{Phen})^3\right]^{2+}$ (Fact)

13. (2) Rate of formation of C_4H_8 is equal to twice the rate of dimerisation of C_2H_4

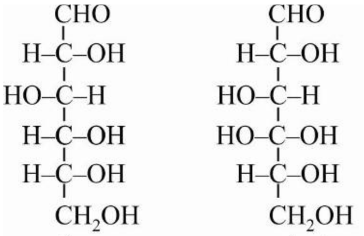
14.(3) Fact

15.(3) $Zn + 2HCl \rightarrow ZnCl_2 + H_2$

$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$

16.(1) Fact

17.(3)



glucose galactose

Configuration of galactose is differ around C-4.

18.(4) $H_3C - CH = CH - CH_3 \xrightarrow[\text{High temp}]{\text{alkaline.KMnO}_4} 2CH_3COOH$

19.(4) $H_3C - CH = CH_2 \xrightarrow{Cl_2/H_2O} H_3C - \underset{\displaystyle OH}{\underset{|}{C}H} - \underset{\displaystyle Cl}{\underset{|}{C}H_2}$

20.(3) Oxidising power is directly proportional to SRP.

21.(3) Fact

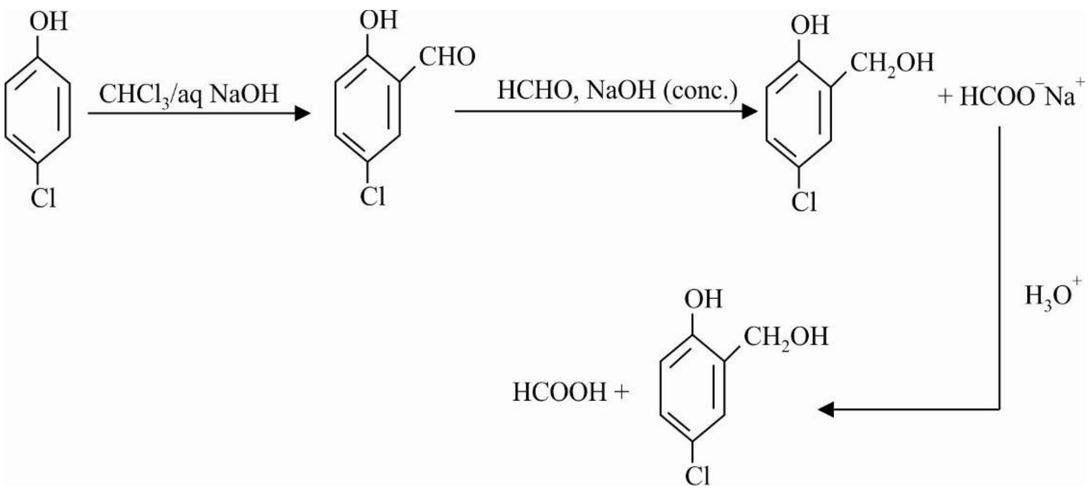
22.(3) Molar mass of $AB_2 = 25 \times 10^{-3}$ kg

Molar mass of $A_2B_2 = 3 \times 10^{-3}$ kg

Molar mass of $A = 5 \times 10^{-3}$ and $B = 10 \times 10^{-3}$

23.(3) RNA is single stranded structure.

24.(3)

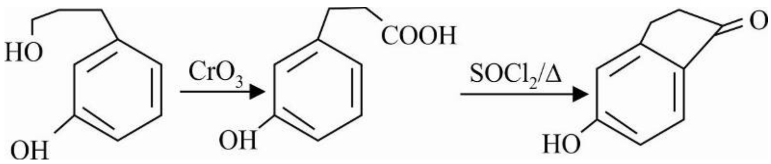


25.(3) $(SiO_4)^{4-}$, orthosilicates

26.(1) Thermal stability of carbonate of alkaline earth metal increases by increasing size of divalent cation.

27.(2) Square planar complex is formed by removing ligand along dz^2 axis. So that its splitting pattern follow second option.

28.(1)



29.(1) pKa value is directly proportional to the –I effect.

30.(1) Peptization is a process of converting freshly prepared ppt into colloidal solution when treated with suitable electrolyte.

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PART-C	MATHEMATICS

1.(2) $S_4 = 2[2a+3d] = 16 \Rightarrow 2a+3d = 8$

$S_6 = 3[2a+5d] = -48 \qquad 2a+5d = -16$

$2d = 24 \Rightarrow d = 12$

$a = 22$

$S_{10} = 5[44-108] = -5\times 64 = -320$

2.(4) $f'(n) = \dfrac{x}{2\sqrt{kn-x^2}}.(k-2n)+\sqrt{kx-x^2} = 0$

$\Rightarrow \quad kx-2x^2+2\left(kx-x^2\right)=0 \quad \Rightarrow \quad 3kx-4x^2=0$

$\Rightarrow \quad x=0$ or $x=\dfrac{3k}{4}=3$

$k=4 \quad \Rightarrow \quad$ Max value of $f(x)=3\sqrt{3}$

3.(2) $(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$

$\left(1-x^2\right)\left(1-x^3\right)^9 = {}^9C_6\left(x^3\right)^6 = \dfrac{9\times 8\times 7}{3\times 2} = 84$

4.(1) $np = 8 \quad npq = 4 \quad q = \dfrac{1}{2} \quad p = \dfrac{1}{2} = n = 16$

$\Rightarrow p(x\le 2) = P(0)+P(1)+P(2) = \dfrac{1}{2^{16}}+\dfrac{16}{2^{16}}+\dfrac{{}^{16}C_2}{2^{16}} = \dfrac{1+16+120}{2^{16}} = \dfrac{137}{2^{16}} = \dfrac{k}{2^{16}} \Rightarrow k = 137$

5.(3) $\left[-\dfrac{1}{3}\right]+\left[-\dfrac{1}{3}-\dfrac{1}{100}\right]+\ldots\left[-\dfrac{1}{3}-\dfrac{66}{100}\right]+\left[-\dfrac{1}{3}-\dfrac{17}{100}\right]-\left[\dfrac{1}{3}-\dfrac{99}{100}\right]$

$-67-2(33) = -67-16 = -133$

6.(4) $\displaystyle\int_0^{\pi/2}\dfrac{\cot x}{\cot x+\mathrm{cosex}} = \int_0^{\pi/2}\dfrac{\cos x}{1+\cos x}dx \int_0^{\pi/2}dx^- \int_0^{\pi/2}\dfrac{1}{1+\cos x}dx$

$$=[x]_0^{\pi/2}-\frac{1}{2}\int_0^{\pi/2}\sec^2\frac{\pi}{2}dx=\frac{\pi}{2}-\frac{1}{2}\left|\frac{\tan\frac{\pi}{2}}{\frac{1}{2}}\right|_0^{\pi/2}=\frac{\pi}{2}-1 \Rightarrow \frac{1}{2}(\pi-2) \Rightarrow M=\frac{1}{2}, n=2$$

$\therefore$ $MN=-1$

7.(4) $P\to(\sim q\vee r)$ $\Rightarrow$ P is T and $\sim q\vee r$ is F now $\sim q\vee r$ $\Rightarrow$ $\sim q$ is False and r is also False.

$\therefore$ q is True r is False.

So, TTF is truth values of $p_1q_1r_1$ respectively.

8.(1) Consider following cases 10 distinct or 9 distinct or 8 distinct … 0 distinct.

$\Rightarrow$ $21_{C_{10}}+{}^{21}C_9+{}^{21}C_8......{}^{21}C_0=S$ (say) … (i)

${}^{21}C_{11}+{}^{21}C_{12}\ldots{}^{21}C_{20}+{}^{21}C_{21}=S$ … (ii) Apply ${}^nC_r={}^nC_{n-r}$

$25=2^{21}$ Adding i and ii

$S=2^{20}$

9.(4) $|z-i|=|z-1|$ is the right bisector of the line joining points $(0,i)$ and $(1,0)$ i.e., $y=x$

10.(4) $3x^2+4y^2=12 \Rightarrow 8y\frac{dy}{dx}=-6x$

$$\frac{dy}{dx}=-\frac{6}{8}\frac{x}{y}$$

$\Rightarrow$ slope of normal $-\frac{dx}{dy}=\frac{4y}{3x}=-2(given)$

$\boxed{2y=-3x}$

Also, x,y lies on $3x^2+4y^2=12$

$$3\left(\frac{2y}{3}\right)^2+4y^2=12$$

$$4y^2+12y^2=36 \Rightarrow 16y^2=36 \Rightarrow y^2=\frac{9}{4}$$

$$y=\pm\frac{3}{2} \Rightarrow x=\mp1\left(\mp1,\pm\frac{3}{2}\right)$$

i.e. $p\left(-1,\frac{3}{2}\right)$ or (4, 4) (A little consideration shows that P must lie in 2nd quadrant)

$$\Rightarrow \sqrt{5^2+\left(\frac{5}{2}\right)^2}=\sqrt{25+\frac{25}{4}}=\frac{5\sqrt{5}}{2}$$

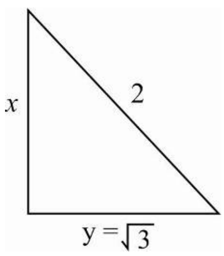
11.(3) $\sin^{-1}\frac{\sqrt{2}}{\sqrt{3}}-\sin^{-1}\frac{3}{5}$

$$=\cos^{-1}\frac{5}{13}-\cos^{-1}\frac{4}{5}=\cos^{-1}\left[\frac{5}{13}\cdot\frac{4}{5}+\frac{12}{13}\cdot\frac{3}{5}\right]-\left(\frac{5}{13}<\frac{4}{5}\right)=\cos^{-1}\left[\frac{56}{65}\right]=\frac{\pi}{2}-\sin^{-1}\frac{56}{65}$$

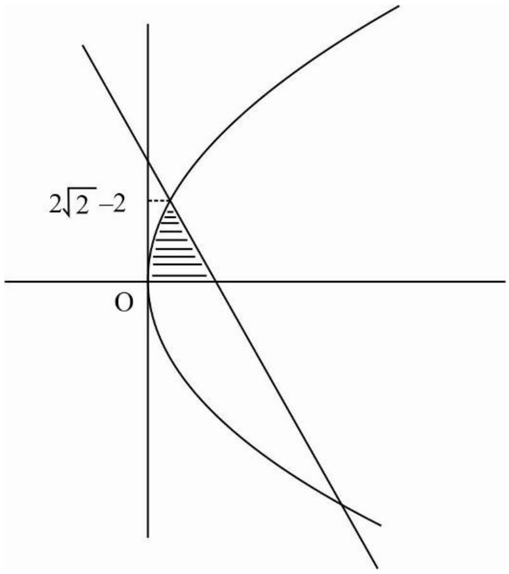
12.(1) $x^2+y^2=4$

$$\frac{dx}{dt}+\frac{2ydy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{x\frac{dx}{dt}}{y}=-\frac{1(25)}{\sqrt{3}}=-\frac{25}{\sqrt{3}}cm/\sec.$$



13.(4) $A = \int_0^{2\sqrt{2}-2} \left(1 - y - \frac{y^2}{4}\right) dy$



$$A = \left[y - \frac{y^2}{2} - \frac{y^3}{12} \right]_0^{2\sqrt{2}-2} = y\left[1 - \frac{y}{2} - \frac{y^2}{12}\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[1 - \left(\sqrt{2}-1\right) - \frac{4}{12}\left(\sqrt{2}-1\right)^2\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[2 - \sqrt{2} - \frac{1}{3}\left(2 + 1 - 2\sqrt{2}\right)\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[2 - \sqrt{2} - 1 + \frac{2\sqrt{2}}{3}\right]$$

$$= 2\left(\sqrt{2}-1\right)\left[1 - \frac{\sqrt{2}}{3}\right] = 2\left(\sqrt{2} - \frac{2}{3} = 1 + \frac{\sqrt{2}}{3}\right)$$

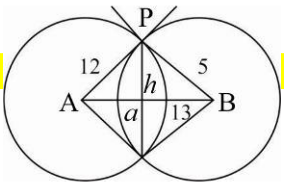
$$= 2\left(\frac{4}{3}\sqrt{2} - \frac{5}{3}\right)$$

14.(2) $A = \dfrac{\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}}{2} = \dfrac{1}{2}\begin{bmatrix} 4 & 8 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$

$$B = \frac{\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}}{2} = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} +4 & -2 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

15.(3)



Area of PAB $= \frac{1}{2} PA \cdot PB = \frac{1}{2} \times h \cdot AB \quad \Rightarrow \quad 12 \times 5 = h \times 13$

$h = \frac{60}{13} \qquad \therefore \qquad$ Length of chord $\frac{120}{13}$

16.(4) $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + k(-8) = \lambda\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

$\left|\lambda(2\hat{i} - 2\hat{j} - k)\right| |3\lambda| = 12 \quad \Rightarrow \quad \lambda = 4 \quad \therefore \quad 8\hat{i} - 8\hat{j} - 4\hat{k}$ is required vector

i.e., $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

17.(2) $\int_0^{f(x)} 4t^3 dt = (x-2)\, g(x)$

Differentiating both sides w.r.t x

$$4\left(f(x)\right)^3 \times f'(x) = (x-2)\, g'(x) + g(x)$$

$\lim\limits_{x\to 2} 4\times 6\times\frac{1}{48}=9(2)\Rightarrow 18$

18.(4) $1+\sin^4 xn=\cos^2 3x$

$\sin^4 x=-\sin^4 3x$

Then solution will exist only if

$\sin^4 x=0$ and $\sin^2 3x=0$

$x=nx$ or $3x=m\pi \quad m_1 n\in z$

i.e., $x=0,\pi,2\pi,-\pi,-2\pi$ are 5 solutions is $\left[\frac{-5\pi}{2},\frac{5\pi}{2}\right]$

19.(4) $\int\frac{2x^3-1}{x^4+x}=\int\frac{2x^3}{x^4+x}-\int\frac{1}{x^4+x}dx=\frac{2}{3}\int\frac{3x^2}{x^3+x}-\int\frac{1}{x.\left(x^3+1\right)}\times\frac{x^2}{x^2}.dx=\frac{2}{3}\log\left(x^3H\right)-\frac{1}{3}\int\frac{dt}{t(t+1)}$

(put $x^3=t$)

$\frac{2}{3}\log\left(x^3+1\right)-\frac{1}{3}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$

$\frac{2}{3}\log\left(x^3+1\right)-\frac{1}{3}\left[\log t-\log(t+1)\right]+c=\frac{2}{3}\log\left(x^3+1\right)-\frac{1}{3}\log\left(\frac{x^3}{1+x^3}\right)+c=\frac{1}{3}\log\left|\left(x^3+1\right)^2\frac{x^3}{1+x^3}\right|+c$

$\frac{1}{3}\log\left|\left(\frac{1+x^3}{x^3}\right)^3\right|+c$

$\log\left|\frac{1+x^3}{x}\right|$

20.(4) $y^2dx+\left(x-\frac{1}{y}\right)dy=0$

$y^2\frac{dx}{dy}+x-\frac{1}{y}=0$

$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$

IF $=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

$\therefore$ Solution is x.IF $=\int\frac{1}{y^3}.e^{-\frac{1}{y}}=xe^{-\frac{1}{y}}=\int -t.e^t dt$ (Put $-\frac{1}{y}=t$)

And apply by parts and replacing t we get

$xe^{-\frac{1}{y}}=+\frac{1}{y}e^{-\frac{1}{y}}+e^{-\frac{1}{y}}+c$

$x=\left(\frac{1}{y}+1\right)+ce^{\frac{1}{y}}$

Given $x=1\ y=1 \quad\Rightarrow\quad C=\frac{-1}{c}$

Now at $y = 2 \quad x = \frac{3}{2} + \frac{1}{e}\left(e^{\frac{1}{2}}\right) = \frac{3}{2} - \frac{1}{\sqrt{e}}$

21.(3) $\frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$

$$\frac{\alpha(1-\beta)+\beta(1-\alpha)}{(1-2)(1-\beta)} = \frac{\alpha+\beta-2\alpha\beta}{1-\alpha-\beta+\alpha\beta} = \frac{\frac{25}{375}-\frac{2(-2)}{375}}{1-\frac{25}{375}-\frac{2}{375}} = \frac{29}{375-27} = \frac{1}{12}$$

22.(3) There are two equilateral ΔS and total number of triangles = ${}^6C_3$

$\therefore \quad \frac{2}{{}^6C_3} = \frac{1}{10}$ is required probability

23.(2) Volume of 11 pipe $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = f(\lambda) \quad (say)$

$f(x) = 1 \qquad \lambda^3 - \lambda + 1$ (to be minimum)

$\therefore \qquad f'(x) = 3\lambda^2 - 1 = 0$

$\lambda = \pm\frac{1}{\sqrt{3}}$

$f''(x) = 6\lambda > at\ \lambda = +\frac{1}{\sqrt{3}} \ \therefore \qquad$ min if $\lambda = \frac{1}{\sqrt{3}}$

24.(3) $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$

$|B| = 5(-1-3) - 2\alpha(-\alpha) + 1(-2\alpha) = -25 + 2\alpha^2 - 2\alpha \qquad \therefore \quad |A| = \frac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$

$\Rightarrow \qquad i.e., 2\alpha^2 - 2\alpha - 24 = 0$

Sum of α : $\alpha + d_2 = 1$

25.(4) Let $y = \frac{1}{2}(2\sin x \cdot \sin(x+2)) - \sin^2(x+1) = \frac{\cos(2)}{2} - \frac{\cos(2x+2)}{2} - \left(\frac{1-\cos(2x+2)}{2}\right) = -\frac{1}{2}[1-\cos 2]$

$y = -\sin^2 1$

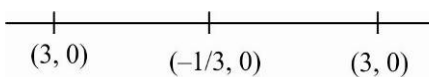
26.(3) $\frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^2 - 8 = \frac{3}{m} \quad$ m = 3 $\frac{y = 3m+1}{x = -\frac{1}{3}}$

$\frac{y = 3m+1}{x = -\frac{1}{3}}$

$e = \sqrt{1 + \frac{8}{1}}$

$e = 3$



$$\frac{3+\frac{1}{3}}{3-\frac{1}{3}}=\frac{10}{8}=\frac{5}{4}$$

27.(2) $e^4+xy=e$

$$\frac{dy}{dx}=\frac{-y}{e^y+x}=\frac{-1}{e} \text{ at } (0,\ 1)$$

$$\frac{d^2y}{dx^2}=-\frac{\left[\left(e^y+x\right)\left(+\frac{dy}{dx}\right)-(y)\left(e^y y^1+1\right)\right]}{\left(e^y+x\right)^2}=-\frac{\left[(e)\left(-\frac{1}{e}\right)-(1)\left[e\left(-\frac{1}{e}\right)+1\right]\right]}{\left(e^4+x\right)^2}=\frac{1}{e^2}$$

28.(4) $\frac{x-2}{3}=\frac{5+1}{2}=\frac{2-1}{-1}=r$

$2+3r,\ -1+2r,\ 1-r$

$2(2+3r)+3(-1+2r)-(1-r)+13=0$

$\underbrace{4+6r-3+6r-1+r+13=0}_{13r+13=0}$

$r=-1 \qquad (-1,\ -3,\ 2)$

$3(2+3r)+(-1+2r)+4(1-r)=16$

$6+\underbrace{9r-1+2r+4-4r}=16$

$7r=7 \qquad (5,1,0)$

$r=1$

$PQ=\sqrt{6^2+4^2+2^2}=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

29.(1) $\frac{x_1+x_2+x_3+x_4}{4}=11 \qquad \frac{x_5+\ldots\ldots\ x_{10}}{6}=16$

$x_1+x_2+\ldots\ldots x_{10}=44+96=140$

X = 14

Variance = $\frac{\Sigma x^2}{4}-\left(\bar{x}\right)^2=\frac{2000}{10}-(14)^2=200-196=4$

30.(3) $\phi\left(\frac{\pi}{3}\right)=hofog(x)=hof\left(\tan\frac{\pi}{3}\right)=hof\left(\sqrt{3}\right)=h\left(3^{1/4}\right)=\frac{1-\sqrt{3}}{1+\sqrt{3}}=\tan\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{12}\right)=\tan\left(\frac{11\pi}{12}\right)$